# Erratum: Femtosecond pulse propagation in nitrogen: Numerical study of (3+1)-dimensional extended nonlinear Schrödinger equation with shock-term correction [Phys. Rev. E 72, 026706 (2005)]

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Numerical precision of the second-order decomposition formula $S_2$ [Eqs. (31) and (32)] in the original paper turned out to be first-order with respect to evolution step $\Delta z$. The second-order decomposition formula that maintains second-order precision is given as follows:

$$S_2(\Delta z) = \hat{U}_K(\Delta z/2)\hat{U}_V(\Delta z)\hat{U}_K(\Delta z/2), \tag{1}$$

where $\hat{U}_K(\Delta z)$ and $\hat{U}_V(\Delta z)$ are second-order symmetric decompositions of partial evolution operators due to the kinetic and interaction parts of the total Hamiltonian, respectively. The partial evolution operators are explicitly written using the discretized exponential operators given in Sec. II A of the original paper:

$$\hat{U}_V(\Delta z) = e^{-i\Delta z\hat{V}_\tau^{(o)}/2}e^{-i\Delta z\hat{V}_\tau^{(e)}/2}e^{-i\Delta z\hat{V}}e^{-i\Delta z\hat{V}_\tau^{(e)}/2}e^{-i\Delta z\hat{V}_\tau^{(o)}/2} \tag{2}$$

$$\begin{aligned}\hat{U}_K(\Delta z) = {} & e^{-i\Delta z\hat{K}_{\tau xx}^{(eo)}/2}e^{-i\Delta z\hat{K}_{xx}^{(o)}/2}e^{-i\Delta z\hat{K}_{\tau xx}^{(oo)}/2}e^{-i\Delta z\hat{K}_{\tau\tau}^{(o)}/2}e^{-i\Delta z\hat{K}_{\tau xx}^{(oe)}/2}e^{-i\Delta z\hat{K}_{\tau yy}^{(oe)}/2}e^{-i\Delta z\hat{K}_{\tau xx}^{(ee)}/2}e^{-i\Delta z\hat{K}_{\tau yy}^{(ee)}/2} \\ & \times e^{-i\Delta z\hat{K}_{\tau\tau}^{(e)}/2}e^{-i\Delta z\hat{K}_{xx}^{(e)}/2}e^{-i\Delta z\hat{K}_{yy}^{(e)}/2}e^{-i\Delta z\hat{K}_{\tau yy}^{(oo)}/2}e^{-i\Delta z\hat{K}_{\tau yy}^{(eo)}/2}e^{-i\Delta z\hat{K}_{yy}^{(o)}}e^{-i\Delta z\hat{K}_{\tau yy}^{(eo)}/2}e^{-i\Delta z\hat{K}_{\tau yy}^{(oo)}/2} \\ & \times e^{-i\Delta z\hat{K}_{yy}^{(e)}/2}e^{-i\Delta z\hat{K}_{xx}^{(e)}/2}e^{-i\Delta z\hat{K}_{\tau\tau}^{(e)}/2}e^{-i\Delta z\hat{K}_{\tau yy}^{(ee)}/2}e^{-i\Delta z\hat{K}_{\tau xx}^{(ee)}/2}e^{-i\Delta z\hat{K}_{\tau yy}^{(oe)}/2}e^{-i\Delta z\hat{K}_{\tau xx}^{(oe)}/2}e^{-i\Delta z\hat{K}_{\tau\tau}^{(o)}/2} \\ & \times e^{-i\Delta z\hat{K}_{\tau xx}^{(oo)}/2}e^{-i\Delta z\hat{K}_{xx}^{(o)}/2}e^{-i\Delta z\hat{K}_{\tau xx}^{(eo)}/2}.\end{aligned} \tag{3}$$

Using Eq. (1), $z$-directional evolution of pulse envelope $\phi$ is calculated as

$$\phi(x,y,z+\Delta z,\tau) = S_2(\Delta z)\phi(x,y,z,\tau) = \hat{U}_K(\Delta z/2)\hat{U}_V(\Delta z)\hat{U}_K(\Delta z/2)\phi(x,y,z,\tau). \tag{4}$$

Here, note that all envelope-dependent coefficients in $\hat{U}_V(\Delta z)$ must be calculated commonly by substituting $\hat{U}_K(\Delta z/2)\phi(x,y,z,\tau)$ into the coefficients, whereas the operand of each discretized exponential operator in Eq. (2) changes due to the preceding operation [1,2].

We performed calculations with the above formula and confirmed that all results in the original paper were at least reproduced visually, although minor numerical differences existed. Among those results, the largest total change of power density distribution was seen in the initial state of the multiple-cone formation [$t$=0 ps in Fig. 3(a)], where interaction terms became large to affect numerical values. However, the change was at most 2.0%, which was reduced to 0.37% in the final power density distribution [Fig. 3(b)].

When performing calculations with the higher-order formula [Eq. (33) in the original paper], we must apply the above formula. Although it appears to include more calculation steps, actual calculation time is similar to the original formula because the increased calculation steps consist of simple operations that do not require much CPU work load. Thus we recommend using the second-order precise formula even when the numerical precision is unimportant.

[1] N. Watanabe and M. Tsukada, Phys. Rev. E **65**, 036705 (2002).
[2] T. Ando, Y. Ohtake, and N. Ohtani, Phys. Rev. E **73**, 066702 (2006).

 